Guide to Mathematics
Released Items:
Understanding Scoring

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1.0 Task Types, Scoring Rubrics, Anchor Sets and Annotations Overview

1.1 Background
The assessment was administered in paper-based and computer-based formats; both modes contained constructed response items. This document is designed to help educators understand the scoring of constructed response items.

1.2 Materials Included in Released Items Packet
- PDF documents containing released item sets with associated materials.
- Documents showing item alignment to the Evidence Statements, answer keys, rubrics and calculator designation.
- Anchor papers with annotations for reasoning and modeling items.
- Additional scored responses that are not annotated.

1.3 Mathematics Task Types
The mathematics items measure critical thinking, mathematical reasoning, and the ability to apply skills and knowledge to real-world problems. Students are asked to solve problems involving the key knowledge and skills for their grade level as identified by the Common Core State Standards (CCSS), using mathematical practices, reasoning, and modeling. You will see examples of reasoning and modeling items in the released item anchor sets. Some items have only one part, and some are composite items that have multiple parts that are related but scored separately.

1.4 Mathematics Scoring Rubrics, Scoring Decisions, Anchor Sets, and Annotations
A scoring rubric, which is created and refined with educator input, describes the level of achievement a response demonstrates for each score point. The Mathematics scoring rubrics are specific to each reasoning and modeling item. The scoring rubric is based on criteria that characterize the level of mathematical understanding as supported by a solution process, modeling, reasoning, explanation, or justification of mathematical principles or concepts.

Each math item has a scoring rubric that describes the score points for that item. The score point range varies by item, and the descriptions of each score point are specific to the unique aspects of the item.

Rubrics include sample student work in order to illustrate what the criteria might look like in a student response and to show a common solution process that the student might use to meet the criteria. Although the rubric example is a common solution, there are many variations of solutions that are correct and can receive the top score point. In addition to scoring rubrics, certain scoring decisions that apply to most or all items have been agreed.

Anchor sets comprise clear and representative examples of student performance at every score point and are created to ensure that scorers interpret rubrics consistently. Anchor set responses include annotations that reference the rubric and examples from the student response in order to explain why the response received a particular score. The anchor set is the primary reference for scorers as they internalize the rubric during training. All scorers have access to the anchor set whenever they are in training or are scoring and are directed to refer to it regularly. Scorers refer to the rubric and compare student responses to those in the anchor set in order to determine the score that best fits the response.
2.0 Scoring Process Overview

2.1 Establishing Standards for Scoring
The scoring standards are established by educators during rangefinding meetings. Educators read student responses and use the rubric criteria to come to consensus of scores. These committee-scored responses are then used to create the scorer training sets.

Scorers are trained to apply only those scoring standards established by the rangefinding committee. This ensures consistency and accuracy in scoring, regardless of who scores the response.

2.2 Scorer Training Process
All scorers must complete training and pass qualification sets prior to scoring student responses. Qualification sets are used to confirm that the scorers have comprehended the scoring criteria and are able to assign the range of scores to student responses accurately.

During training, scorers review the item to which they are assigned and the associated scoring rubric. Next, scorers review the anchor set and annotations that explain the rationale for the score point that was assigned to each response.

After a thorough review of the anchor set responses, practice sets are used to help scorer trainees develop experience using the rubric and anchor sets to score student responses. Practice sets provide guidance and practice for trainees in defining the line between score points, as well as in applying the scoring criteria to a wider variety of responses. Some practice responses clearly reinforce the scoring guidelines presented in the anchor set, while others are selected because they are more difficult to evaluate and/or represent unusual approaches to the task.

After completing practice sets, scorer trainees take qualification sets that are used to confirm that trainees have comprehended the scoring criteria and are able to assign the range of scores to student responses accurately. The responses in these sets are selected to clearly represent the scoring criteria illustrated in the anchor sets. Scorer trainees must demonstrate acceptable performance on these sets by meeting a pre-determined standard for accuracy in order to qualify to score the assessment. As scorers continue scoring, there are additional quality checks built in throughout the process.
3.0 Released Items

3.1 Using Released Items
This item release contains several reasoning (Sub-Claim C) and modeling (Sub-Claim D) items for each grade and course. The question and item-specific scoring rubric are provided for each item. Anchor papers for each score point are included for each of the reasoning and modeling items. These responses, which are grouped by score point, represent a range of approaches that students take when responding to each reasoning and modeling item in mathematics. All student responses appear as the students wrote them. Anchor papers do not represent every possible way to achieve each score point. There can be many ways to achieve a particular score point.

Practice Responses
All tasks include annotations for the anchor sets as well as a set of responses that can be used to practice scoring with the rubric and anchor set responses.
Anchor and practice responses are available.

3.2 Mathematics Scoring Decision Examples
As you read through the reasoning and modeling items in each grade, you will want to be aware of some general scoring decisions. The following are released items or parts of released items with sample student responses. These demonstrate some general scoring decisions that are made during the rangefinding and scoring process. General scoring decisions are applied throughout all types of student responses.
General Scoring Decision: If an item does not specify a method, the student can use any valid method, even if it is not specified in the sample student response of the rubric.

Suppose that \( y = 2x - 3 \). The following points lie on the graph of this equation:

\[
A(a, 2a - 3) \quad B(b, 2b - 3) \quad C(c, 2c - 3)
\]

Amy claims that the slopes of \( \overline{AB} \), \( \overline{BC} \) and \( \overline{AC} \) are equal. Prove that Amy’s claim is correct. Show your work and explain your reasoning.

Enter your answer, your work, and your explanation in the space provided.

\[
\frac{(2b - 3) - (2a - 3)}{b - a} = \frac{2b - 3 - 2a + 3}{b - a} = \frac{2b - 2a + (-3) + 3}{b - a} = \frac{2b - 2a}{b - a} = \frac{2(b - a)}{b - a} = 2
\]
<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
</table>
| 2     | **BC**: \[
\frac{(2c-3)-(2b-3)}{c-b} = \frac{2c-3-2b+3}{c-b} \\
= 2c-2b+(3)-(-3) \\
= 2c-2b+3 \\
= 2c-2b \\
= \frac{2(c-b)}{c-b} = 2
\]  
**AC**: \[
\frac{(2c-3)-(2a-3)}{c-a} = \frac{2c-3-2a+3}{c-a} \\
= 2c-2a+(3)-(-3) \\
= 2c-2a+3 \\
= 2c-2a \\
= \frac{2(c-a)}{c-a} = 2
\]  
Each ratio equals 2. Amy is correct. All the slopes are equal.  
Note: A student can also receive a score of 2 by demonstrating a complete understanding, using appropriate mathematical reasoning/work. |
| 1     | Student response includes 1 of the 2 elements.  
Note: A student can also receive a score of 1 by demonstrating a partial understanding, using appropriate mathematical reasoning/work. |
| 0     | Student response is incorrect or irrelevant. |
Scoring Decision Application: The sample student response in the rubric of this Algebra 1 item refers to students calculating the slope between each of the three points algebraically and providing the correct explanation. There is a note in the rubric to show a student response may receive credit for an alternate approach that demonstrates a complete understanding of slope.

The item gives an equation of a line and states that the three points are on the graph of the equation. This anchor paper shows a student who understands how points on a line relate to slope between those two points and does this without calculating the specific slope like the sample student response. This student still earns full credit of a 2.
General Scoring Decision: If an item specifies a method, the student must use this method to earn full credit.

Explain how to find $2 \times \frac{5}{12}$ using the number line.

Find the product.

![Number line diagram]

**M00778 Rubric**

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Student response includes the following 3 elements.</td>
</tr>
</tbody>
</table>

- **Reasoning component** = 2 points
  - Valid explanation of how to find $\frac{5}{12}$ using the number line
  - Valid explanation of how to find $2 \times \frac{5}{12}$ using the number line

- **Computation component** = 1 point
  - Correct product, $\frac{10}{12}$ or equivalent

Sample Student Response:

I know that each tick mark on this number line is equivalent to $\frac{1}{12}$, so to find $\frac{5}{12}$, I would count 5 of the tick marks.

Then to find $2 \times \frac{5}{12}$, I would count $\frac{5}{12}$ two times starting at zero on the number line. I would land on $\frac{5}{6}$, which is the same as $\frac{10}{12}$. The product is $\frac{10}{12}$.

Note: Student responses must provide explanations to receive the reasoning component points. Simply identifying the locations of 5/12 and 10/12 is not sufficient for reasoning credit.

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Student response includes 2 of the 3 elements.</td>
</tr>
<tr>
<td>1</td>
<td>Student response includes 1 of the 3 elements.</td>
</tr>
<tr>
<td>0</td>
<td>Student response is incorrect or irrelevant.</td>
</tr>
</tbody>
</table>
Scoring Decision Application: This Grade 4 item, unlike the previous Algebra 1 item, asks the student to use a specific method. Because of this, the rubric requires the use of the number line. In most items, students are allowed and even encouraged to use multiple methods to solve an item. However, in order to determine if a student has fully demonstrated the rigor spelled out in the Evidence Statements based on the CCSS, some items need to specify a method. The Evidence Statement for this item states that students must "base explanations/reasoning on a number line diagram". Looking at the anchor paper the student did not earn full credit because in the process of multiplying the two numbers, they did not explain using the number line.
General Scoring Decision: If a student makes a mistake early on in the problem and their work follows that single mistake, they do not get deducted further points.

Two Web sites launched on the same day. At the end of the first week, the number of visitors to each Web site was 25. For the first eight weeks, the number of visitors to each Web site increased according to the corresponding rules.

Web site A: The number of visitors doubled each week.

Web site B: The number of visitors increased by 150 each week.

Part A

Complete the table to show the number of visitors to each Web site for the first eight weeks.

Enter your answers in the table.

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
<th>Week 6</th>
<th>Week 7</th>
<th>Week 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Part B

Based on the data for the first eight weeks, Jose claims that the number of visitors to each Web site can be modeled as a linear function of the number of weeks online. For each Web site, decide if Jose’s claim is correct. If it is correct, explain why. If it is not correct, explain why and describe a more appropriate model.

Enter your answers and your explanations in the space provided.
### VF650047 Rubric Part A

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
</table>
| 1     | Student response includes the following element.  
  - **Computation component** = 1 point  
    
    Web Site A | Web Site B |
    Week 1: 25 | 25         |
    Week 2: 50 | 175        |
    Week 3: 100| 325        |
    Week 4: 200| 475        |
    Week 5: 400| 625        |
    Week 6: 800| 775        |
    Week 7: 1,600| 925   |
    Week 8: 3,200| 1,075    |

Note: All values must be correct to earn the point.

| 0     | Student response is incorrect or irrelevant. |

### VF650047 Rubric Part B

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
</table>
| 2     | Student response includes the following 2 elements.  
  - **Reasoning component** = 2 points  
    - Valid reasoning about the nature of the data for Web Site A  
    - Valid reasoning about the nature of the data for Web Site B  

Sample Student Response:

Jose's claim is not correct about Web Site A. A linear model assumes that the rate of change is constant for each one-week increment. Looking at the data in the table, the change from Week 1 to Week 2 is 25, the change from Week 2 to Week 3 is 50, and the change from Week 3 to Week 4 is 100. The change in visitors from one week to the next is not constant; the change in visitors from one week to the next is two times the change in the previous week. Such data are better suited for an exponential model.

Jose's claim is correct about Web Site B. The rate of change in the number of visitors from one week to the next is constant. The change from one week to the next is 150 visitors each week.

| 1     | Student response includes 1 of the 2 elements. |
| 0     | Student response is incorrect or irrelevant. |
Part A

<table>
<thead>
<tr>
<th>Week 1:</th>
<th>Week 2:</th>
<th>Week 3:</th>
<th>Week 4:</th>
<th>Week 5:</th>
<th>Week 6:</th>
<th>Week 7:</th>
<th>Week 8:</th>
</tr>
</thead>
</table>

Part B

Web Site A: yes, because it goes up at a constant rate.

Web Site B: no, because it doubles each time which is not at a constant rate. A more appropriate model would be an exponential function.

**Scoring Decision Application:** In this Algebra 1 item, students are asked to construct a table in Part A and then use this information to answer the problem in Part B.

In the anchor paper, the student incorrectly filled in the table by switching the columns. The student did not earn credit for Part A. However, since the student then answered Part B correctly based on the information in the table in his or her Part A, he or she did not lose any additional points.
General Scoring Decision: In order to earn the top score point, a student must demonstrate the precision spelled out in the CCSS Practice Standard 6 "attend to precision".

Use the information provided to answer Part A and Part B.

Maria bought wood, paper, and string to make one kite. The list shows the amount and the unit cost of each item she bought.

- 12 square feet of paper at $1 per square foot
- 4 feet of wood at $3 per foot
- 14 yards of string at $2 per yard

**Part A**

What was the total cost of the items Maria bought? Show all the steps you took to find your answer. Be sure to label your answer.

**Part B**

Maria will make 4 more kites for her friends. Determine how much paper, wood, and string are needed and the total cost to make the 4 kites. Show all the steps you took to find your answer. Be sure to label your answer.

<table>
<thead>
<tr>
<th>VF646620 Rubric Part A</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Score</strong></td>
</tr>
</tbody>
</table>
| 3 | Student response includes the following 3 elements.  
  - **Computation component** = 1 point  
    - Correct total cost of the items Maria bought, $52  
  - **Modeling component** = 2 points  
    - The student shows all of the steps in finding the total cost.  
  Sample Student Response:  
  $52
  
  The cost for each item is:  
  Paper: 12 x $1 = $12  
  Wood: 4 x $3 = $12  
  String: 14 x $2 = $28  
  Notes:  
  - Multiplication does not need to be shown as equations.  
  - If a multiplication error occurs, the computation component is not correct. If all three multiplications are wrong, the point comes off the computation component, not from the modeling component. If the student shows three multiplication problems with correct factors, credit can be given for 1 modeling point, even though all three may have computation errors.  
  - $12 + $12 + $28 = $52 |
### VF646620 Rubric Part A

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
</table>
| 3     | Notes:  
  - Addition does not need to be shown as an equation.  
  - If an addition error occurs, the computation component is not correct, but the student can still receive credit for 1 modeling point if the correct addends are used in the model.  

A single equation can be shown for both modeling and computation parts, such as:  12 x $1 + 4 x $3 + 14 x $2 = $52$. However, the answer must have context in terms of money. If no dollar sign appears with the final answer, then no computation point can be given. |
| 2     | Student response includes 2 of the 3 elements. |
| 1     | Student response includes 1 of the 3 elements. |
| 0     | Student response is incorrect or irrelevant. |

### VF646620 Rubric Part B

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
</table>
| 3     | Student response includes the following 3 elements.  
  - **Computation component** = 2 points  
    - Correct amount of paper, wood, and string needed for the 4 kites  
    - Correct total cost for the 4 kites  
  - **Modeling component** = 1 point  
    - The student shows all of the steps in finding the answers.  

Sample Student Response:  

48 square feet of paper, 16 feet of wood, 56 yards of string  

$208  

Amount of Paper: 12 x 4 = 48  

Amount of Wood: 4 x 4 = 16  

Ammount of String: 14 x 4 = 56  

Cost of Paper: 48 x 1 = 48  

Cost of Wood: 16 x 3 = 48  

Cost of String: 56 x 2 = 112  

48 + 48 + 112 = 208  

Or any other valid process.  

Note: A correctly computed total cost in Part B that is based on an incorrect cost in Part A should receive credit.  

| 2     | Student response includes 2 of the 3 elements. |
| 1     | Student response includes 1 of the 3 elements. |
| 0     | Student response is incorrect or irrelevant. |
Scoring Decision Application: This Grade 5 item refers to square feet, feet, yards, dollars and unit costs. In order to earn full credit, the student must label all parts of his or her answer appropriately. In this anchor paper, the student did not appropriately label all parts of the answer, specifically the $208. The student could not receive the top score point because of this omission. If the student labeled his or her answer correctly, this would have earned the top score point of 6. Due to this precision error, one point is deducted and the student response is scored a 5. If the student had already lost a point in Part A or B due to a computation or another type of error, an additional point would not have been deducted for precision.
Other ways a student can lose a precision point include incorrect definition of variables, using an undefined variable, using run-on equations (i.e. $24 \times 16 = 384 \div 4 = 96$), using incorrect mathematical vocabulary (i.e. referring to an inequality as an equation), etc.

Examples of this scoring decision can be seen in:

- Grade 6 VF888578 Anchor Paper 3 (2015)
- Grade 7 VF654249 Anchor Paper 4 (2015)
- Grade 4 M03520 Anchor Paper 6 (2016)
- Grade 6 VH061163 Anchor Paper 4 (2016)
- Algebra II VF886949 Anchor Paper 2 (2016)