Math
Spring Operational 2015

Integrated Mathematics III
Performance Based Assessment
Released Items

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1. The table shows the steps and explanations that can be used to solve $\sqrt{x} - 5x = -4$.

	Work	Explanation
	$\sqrt{x} - 5x = -4$	Given
Step 1	$\sqrt{x} = 5x - 4$	Addition property of equality
Step 2	$x = 25x^2 - 40x + 16$	Square both sides of the equation
Step 3	$0 = 25x^2 - 41x + 16$	Subtraction property of equality
Step 4	$0 = (25x - 16)(x - 1)$	Factor
Step 5	$(25x - 16) = 0$ or $(x - 1) = 0$	Zero product property
Step 6	$25x = 16$ or $x = 1$	Addition property of equality
Step 7	$x = \frac{16}{25}$ or $x = 1$	Division property of equality

Which step in the table could have created an extraneous solution?

- A. Step 1
- B. Step 2
- C. Step 4
- D. Step 5

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2. Which equations are true for all values of x?

Select **all** that apply.

- ☐ A. $3^{2-x} = 3^2 - 3^x$
- ☐ B. $3^{x+2} = 9\,(3^x)$
- ☐ C. $(3^x)^2 = (3^2)^x$
- ☐ D. $9^{x+2} = 3^{2x+4}$
- ☐ E. $27^x = (3^x)^3$

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3. For what value of m is the equation true?

$$x^2 + 10x + 11 = m + (x + 5)^2 - 25$$

Enter your answer in the box.

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4. Functions f and g are defined as $f(x) = 2^x$ and $g(x) = x + 3$.

What are the values of x to the nearest hundredth when $f(x) = g(x)$?

Enter your answers in the boxes.

$x_1 =$ ☐

$x_2 =$ ☐

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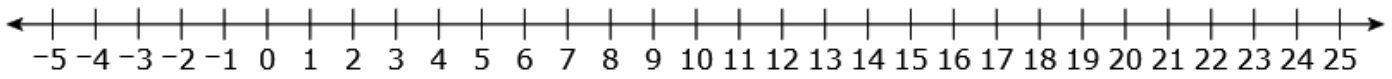
5. Point A is located at -3, and point B is located at 19.

Select a point on the number line between A and B such that the distance from A to the point is $\frac{3}{11}$ of the distance from A to B.

Select a place on the number line to plot the point.



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6. To prepare for a test, three students have been asked to present a review lesson to their class on sketching the graph of a parabola in the *xy*-coordinate plane. They decide to use the quadratic function $f(x) = 4x^2 + 8x - 5$ in their presentation. Each student will use algebra to explain how to find one of three key features of the graph.

- Angella rewrites the equation in factored form.
- Benjamin rewrites the equation by completing the square.
- Carla evaluates $f(0)$.

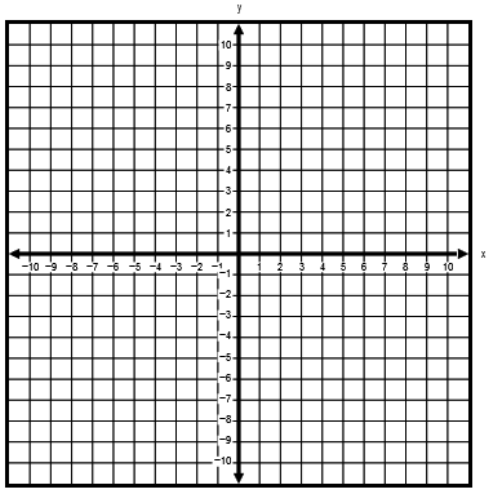
Part A

Sketch the graph of the function on the *xy*-coordinate grid shown.

1. Select the quadratic button.

2. Drag the vertex and another point to graph the function.

Quadratic



Part B

Describe how each student's work contributes to finding the key features of the graph. Complete their work and describe the key feature that is revealed.

Enter your descriptions and your work in the space provided.

[A] [π]

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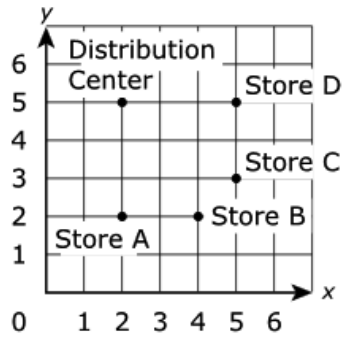
7. The arrangement of a distribution center and four stores to which it delivers is shown on the grid. Each unit on the grid represents 5 miles. The grid lines represent the roads.



The distribution center operators will use a single vehicle and must decide between a large truck and a small van. They will base their decision on this information.

Large Truck

- fuel efficiency: 9 miles per gallon
- delivers to all stores in one round-trip
- uses the shortest route to go to Stores A through D and then back to the distribution center
- must pay a $3.00 fee at each store for using the loading dock

Small Van

- fuel efficiency: 18 miles per gallon
- must return to the distribution center after delivery to each store
- uses the shortest route between the distribution center and each store
- no fee for using the loading dock

Part A

Create a model that shows the total cost to deliver to all the stores for each type of vehicle based only on the information provided. Justify your models, including any assumptions you made.

Enter your models and your justifications in the space provided.

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Part B

Determine when it is more cost efficient to use the van for deliveries than it is to use the truck. Justify your answer.

Enter your answer and your justification in the space provided.

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8. A student claims that there is no solution to the system of inequalities shown.

$$\begin{cases} y \geq x^2 + 3 \\ y < \frac{x}{2} + 1 \end{cases}$$

- Explain a method for proving that the student's claim is correct.
- Identify a single change that could be made to the system of inequalities so that it does have a solution set. Explain your answer.
- Give an ordered pair that would be part of the solution that results from your change.

Enter your answer and your justification in the space provided.

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9. An online bookstore that sells paperback books is changing its shipping pricing from a weight-based fee of $0.18 per ounce to a per-item flat fee.

Part A

The manager of the bookstore created a frequency table showing the weights of the different paperback books and the number of paperback books sold at each weight for one week.

Weight (to the nearest ounce)	Number of Books Sold
10	1
11	2
12	3
13	4
14	6
15	3
16	1
17	2
18	1
19	1

Determine a reasonable per-item flat fee the bookstore can charge so that the flat-fee is comparable to the original $0.18 per ounce weight-based fee. Justify your reasoning.

Enter your answer and your justification in the space provided.

[A] [π]

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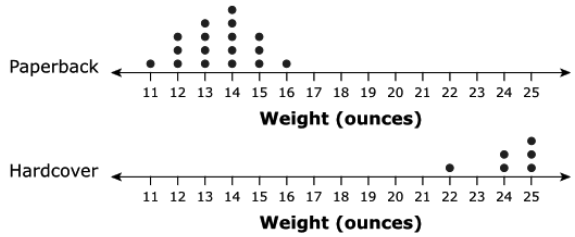
Part B

The manager decides to ship hardcover books in addition to the paperback books. The dot plots show the number of books for each weight shipped for a second week.



Based on the data shown in the dot plots, what new pricing plan would you use? Justify your reasoning.

Enter your answer and your justification in the space provided.

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10. Enrique is recording the number of hamburgers and hot dogs that he eats each week. The table shows the nutritional information for hot dogs and hamburgers that he found on the Internet.

	Calories	Fat (in grams)
Hot Dog	240	16
Hamburger	300	10

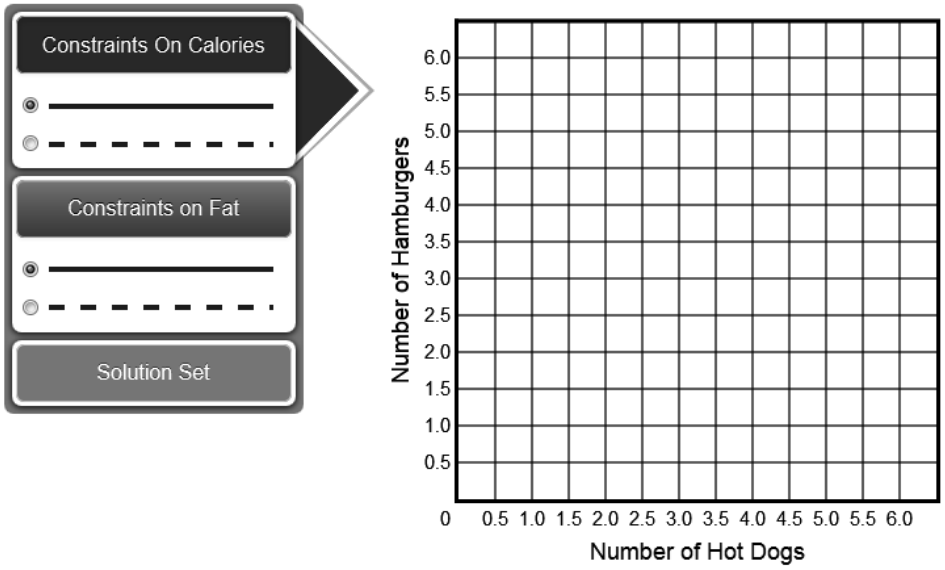
From the hamburgers and hot dogs Enrique eats each week, he decides to have no more than 1,200 total calories and no more than 60 total grams of fat.

Part A

On the coordinate plane, graph the two lines that represent the maximum number of hot dogs and hamburgers Enrique can eat in one week due to the constraints on calories and fat.

Graph the solution set by

- selecting the "Constraints on Calories" button to graph the line and choosing the line style,
- selecting the "Constraints on Fat" button to graph the line and choosing the line style, and then
- selecting the "solution set" button to select the desired region.



Part B

Give an example of one point that satisfies the constraints and one that does not satisfy the constraints. Interpret the meaning in the context of the situation being modeled. Explain your answer.

Enter your explanation in the space provided.

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11. **Part A**

A radio station's broadcast signals were able to reach within a 20-mile radius of the location of its radio tower. The owners of the radio station hired engineers to work on the radio tower and strengthen these signals so that the radio station would be able to reach an area with a greater radius.

Create a mathematical model to find the difference between the area that the broadcast signals could previously reach and the area that the broadcast signals can now reach, in terms of the number of miles the radius increased.

Enter your answer in the space provided.

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Part B

The area in which the radio station broadcasts has a population density of about 74 people per square mile.

After the radio station strengthened its broadcast signals, it was able to reach a total population of approximately 160,000 people. By how many miles did the radio station increase its broadcast radius? Round your answer to the nearest hundredth of a mile. Show your work and explain your answer.

Enter your answer, your work, and your explanation in the space provided.

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12. **Part A**

Let x and y represent natural numbers. Prove that the following equation is true for all x and y values. Show your work or explain your answer.

$$\left(x^2+y^2\right)^2-\left(x^2-y^2\right)^2=(2xy)^2$$

Enter your work and your explanation in the space provided.

Part B

Jenna claims that x^2-y^2, $2xy$, and x^2+y^2 can be used to find side lengths for right triangles. Explain why Jenna is correct. Also, explain what restrictions, if any, must be placed on the values of x and y when they are being used to find side lengths for right triangles.

Enter your explanations in the space provided.

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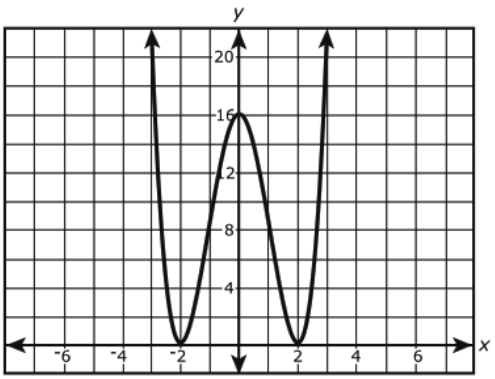
13. **Part A**

The graph $y = P(x)$ is shown.



What property of an even function do you see in this graph?

Enter your answer in the space provided. Enter **only** your answer.

[A] [π]

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Part B

An equation for $y = P(x)$ is shown.

$$y = P(x) = x^4 - 8x^2 + 16$$

State the algebraic definition of an even function. Show algebraically that $P(x)$ is an even function for all values of x.

Enter your answer in the space provided. Enter **only** your answer.

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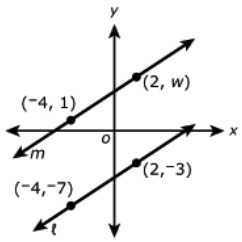
14. **Part A**

Line ℓ passes through the points $(-4, -7)$ and $(2, -3)$ on the coordinate plane.
Line m passes through the points $(-4, 1)$ and $(2, w)$.

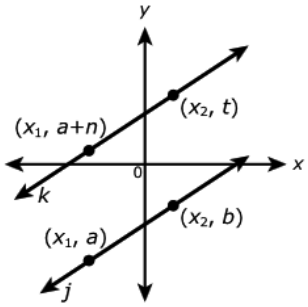


For what value of w is line m parallel to line ℓ?

Enter your answer in the box.

Part B

Given the figure, write an expression that can replace t and will guarantee that lines j and k are parallel.
Support your answer.



Enter your expression and your support in the space provided.

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