

Math
Released Item 2019

Geometry

Diagram
M41165

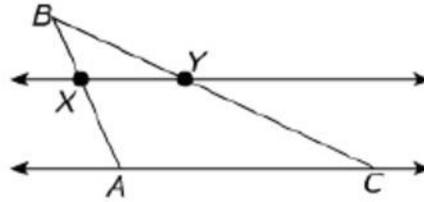
Anchor Set
A1–A9

With Annotations

Prompt

In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.



Prove that \overline{XY} and \overline{AC} are parallel.

Rubric

#10 M41165 Rubric

Score	Description
3	<p>Student response includes each of the following 3 elements.</p> <ul style="list-style-type: none"> • Reasoning component = 1 point • Reasoning component = 1 point <ul style="list-style-type: none"> ○ Correct reasoning that accurately uses geometric relationships and properties of lines, segments, angles, and triangles to prove that \overline{XY} and \overline{AC} are parallel (2 points) • Modeling component = 1 point <ul style="list-style-type: none"> ○ Correct use of notation and vocabulary to support reasoning <p>Sample Student Response:</p> <p>$\triangle XBY$ and $\triangle ABC$ are similar triangles. This follows from the SAS Triangle Similarity criterion.</p> $\angle B \cong \angle B \qquad \frac{BX}{BA} = \frac{1}{3} \qquad \frac{BY}{BC} = \frac{1}{3}$ <p>Therefore, $\angle BXY \cong \angle BAC$.</p> <p>If corresponding angles resulting from two lines cut by a transversal are congruent, then the lines are parallel. Therefore, \overline{XY} and \overline{AC} are parallel.</p> <p>Note: When a student references Angle Y, scorers are to assume Angle BYX, and when a student references Angle X, scorers are to assume Angle BXY.</p>
2	Student attempts to write a logical proof and works with similar triangles but makes mistakes in notation, logic, and steps in justification.
1	Student attempts to write a proof but does not show much ability to write a logical argument with reasons or justification.
0	The response is incorrect or irrelevant.

Statements	Reasons
$BX = \frac{1}{3}(BA)$ $BY = \frac{1}{3}(BC)$	Given
$\frac{BX}{BA} = \frac{BY}{BC}$	Proportions - Substitution ($\frac{1}{3} = \frac{1}{3}$)
$\angle B \cong \angle B$	Reflexive Property
$\triangle BAC \sim \triangle BXY$	SAS Similarity
$\angle BXY \cong \angle BAC$	Corresponding Angles of Similar Triangles are Congruent
$\overline{XY} \parallel \overline{AC}$	If corresponding angles made by 2 lines cut by a transversal are congruent, then the lines are parallel

Annotation

Anchor Paper 1 **Score Point 3**

The response receives full credit. The student shows complete reasoning.

- The response provides complete reasoning that accurately uses geometric relationships and properties of lines, segments, angles and triangles to prove that \overline{xy} and \overline{ac} are parallel ($BX=1/3(BA)$, $BY=1/3(BC)$ - Given; $BX/BA = BY/BC$ - Proportions - Substitution ($1/3=1/3$); $\angle B \cong \angle B$ - Reflexive Property; $\triangle BAC \sim \triangle BXY$ - SAS Similarity; $\angle BXY \cong \angle BAC$ - Corresponding Angles of Similar Triangles are congruent; $\overline{xy} \parallel \overline{ac}$ - If corresponding angles made by 2 lines cut by a transversal are congruent, then the lines are parallel).

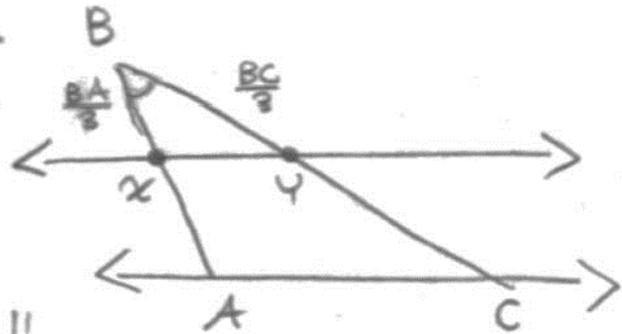
Given: B, X, A collinear

B, Y, C collinear

$$BX = \frac{1}{3} BA$$

$$BY = \frac{1}{3} BC$$

Prove $\overleftrightarrow{XY} \parallel \overleftrightarrow{AC}$



① Given

② $\angle B \cong \angle B$

③ Given

④ $\frac{BX}{BA} = \frac{1}{3}$

$$\frac{BY}{BC} = \frac{1}{3}$$

⑤ $\frac{BX}{BA} = \frac{BY}{BC}$

⑥ $\triangle BXY \sim \triangle BAC$

⑦ $\angle X \cong \angle A$

⑧ $\overleftrightarrow{XY} \parallel \overleftrightarrow{AC}$

① Given

② Reflexive

③ Given

④ Prop of proportions

⑤ Trans prop =

⑥ SAS \sim

⑦ corres $\angle s \sim \Delta s \cong$

⑧ corres $\angle s \cong \Rightarrow \parallel$ lines

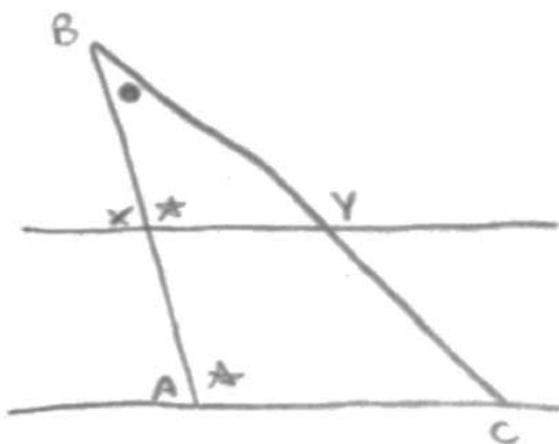
Annotation

Anchor Paper 2 Score Point 3

The response receives full credit. The student shows complete reasoning.

- The response provides complete reasoning that accurately uses geometric relationships and properties of lines, segments, angles and triangles to prove that \overleftrightarrow{xy} and \overleftrightarrow{ac} are parallel (1. Given: B,X,A collinear, B,Y,C collinear; 1. Given; 2. $\angle B \cong \angle B$; 2. Reflexive; 3. $BX = \frac{1}{3}BA$, $BY = \frac{1}{3}BC$; 3. Given; 4. $\frac{BX}{BA} = \frac{1}{3}$, $\frac{BY}{BC} = \frac{1}{3}$; 4. Prop of proportions; 5. $\frac{BX}{BA} = \frac{BY}{BC}$; 5. Trans prop =; 6. $\triangle BXY \sim \triangle BAC$; 6. SAS \sim ; 7. $\angle X \cong \angle A$; 7. corres \angle 's $\sim \triangle \cong$; 8. $\overleftrightarrow{xy} \parallel \overleftrightarrow{ac}$; 8. corres \angle 's $\cong \implies \parallel$ lines.

S	R
<p>① B, X, A are collinear; B, Y, C are collinear; $BX = \frac{1}{2}(BA)$ and $BY = \frac{1}{2}(BC)$</p>	① given
② $\frac{BX}{BA} = \frac{BY}{BC}$	② transitive
③ $\sphericalangle B = \sphericalangle B$	③ reflexive
④ $\triangle BXY \sim \triangle BAC$	④ SAS \sim
⑤ $\sphericalangle BXY = \sphericalangle BAC$	⑤ def. of \sim
⑥ $XY \parallel AC$	⑥ = corresponding \sphericalangle 's \rightarrow // lines



Annotation

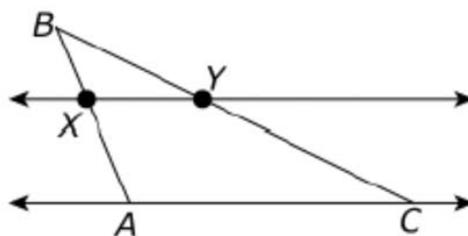
Anchor Paper 3 Score Point 3

The response receives full credit. The student shows complete reasoning.

- The response provides complete reasoning that accurately uses geometric relationships and properties of lines, segments, angles and triangles to prove that \overline{xy} and \overline{ac} are parallel (1. B,X,A are collinear, B,Y,C are collinear, $BX=1/3(BA)$ and $BY=1/3(BC)$; 1. Given; 2. $BX/BA=BY/BC$; 2. Transitive; 3. $\angle B = \angle B$; 3. Reflexive; 4. $\triangle BXY \sim \triangle BAC$; 4. SAS \sim ; 5. $\angle BXY = \angle BAC$; 5. def. of \sim ; 6. $XY \parallel AC$; 6. = corresponding \angle 's $\rightarrow \parallel$ lines.

In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.



Prove that \overleftrightarrow{XY} and \overleftrightarrow{AC} are parallel.

$B, X,$ and A are collinear; $B, Y,$ and C are collinear;
 $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.

Given

$$\angle ABC \cong \angle XBY$$

Reflexive Property

$$\triangle ABC \sim \triangle XBY$$

Side Angle Side

$$\angle BXY \cong \angle BAC$$

Corresponding angles of similar triangles

Annotation

Anchor Paper 4 Score Point 2

The response receives partial credit. A logical proof is written with mistakes in logic and steps in justification.

- The response has a written logical proof with a missing step (B,X, and A are collinear, B,Y and C are collinear, $BX=1/3(BA)$ and $BY=1/3(BC)$; Given; $\angle ABC \cong \angle XBY$; Reflexive Property; $\triangle ABC \sim \triangle XBY$; Side Angle Side; $\angle BXY \cong \angle BAC$; Corresponding angles of simmilar triangles).

The response provides a written logical proof and reasoning but is missing the final step showing and justifying that \overline{xy} and \overline{ac} are parallel. This results in an overall score of 2 for this response.

Statement	Reason
1. B, X, C are collinear B, Y, C are collinear $BX = \frac{1}{3}(BA)$ $BY = \frac{1}{3}(BC)$	1. Given
2. $\frac{BX}{BA} = \frac{1}{3}, \frac{BY}{BC} = \frac{1}{3}$	2. Division
3. $\frac{BX}{BA} = \frac{BY}{BC}$	3. Transitive
4. \overleftrightarrow{XY} and \overleftrightarrow{AC} are parallel	4. Side Splitter Theorem - if a line divides two sides of a triangle so that those two divided segments are proportionally divided, the lines are parallel

Annotation

Anchor Paper 5 Score Point 2

The response receives partial credit. A logical proof is written with mistakes in logic and steps in justification.

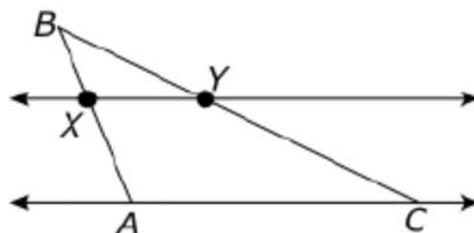
- The response includes a logical proof with a transcription error that detracts from the responses overall score (1. B, X, +C are collinear, B, Y, +C are collinear, $BX = \frac{1}{3}(BA)$, $BY = \frac{1}{3}(BC)$; 1. Given; 2. $BX/BA = \frac{1}{3}$, $BY/BC = \frac{1}{3}$; 2. Division; 3. $BX/BA = BY/BC$; 3. Transitive; 4. \overline{xy} and \overline{ac} are parallel; 4. Side Splitter Theorem – if a line divides two sides of a triangle so that those two divided segments are proportionally divided, the lines are parallel).

Note: The first line under the statement heading is incorrectly transcribed. It should say that B, X and **A** are collinear. All of the other reasons under the statement heading are correct. This transcription error causes a one point deduction for precision because this response would otherwise be a top score paper.

Since the Side Splitter Theorem used in this response is a correct alternative way to prove the lines are parallel, the part of the proof showing the triangles are similar is not required and could receive a top score for a complete and correct proof.

In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.



Prove that \overleftrightarrow{XY} and \overleftrightarrow{AC} are parallel.

$$BX = \frac{1}{3} (BA) - \text{GIVEN}$$

$$BY = \frac{1}{3} (BC) - \text{GIVEN}$$

$$\triangle BXY \approx \triangle BAC - \text{CPCTS}$$

$$\overleftrightarrow{XY} \parallel \overleftrightarrow{AC}$$

Annotation

Anchor Paper 6 Score Point 1

The response earns partial credit. A proof is attempted but does not show logical arguments with reasons and justifications.

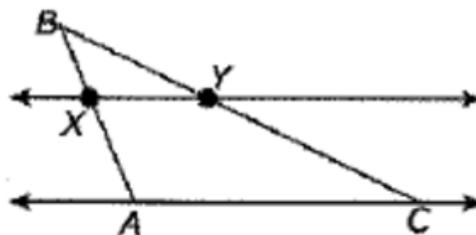
- The response shows an attempt at a proof with no reason or justification ($BX = 1/3(BA)$ - GIVEN; $BY = 1/3(BC)$ - GIVEN; $\triangle BXY \approx \triangle BAC$ - CPCTS; $\overline{XY} \parallel \overline{AC}$).

The student provided the givens from the prompt in the proof then gave the next step showing the triangles are similar ($\triangle BXY$ is similar to $\triangle BAC$) but has a missing step showing the angle B is equal to angle B by the reflexive property. Also the reason of CPCTS is incorrect since the triangles are not congruent. The proof stops short of finding equal angles, which is the step needed to prove the lines are parallel.

The use of the approximation symbol (\approx) instead of a similarity symbol (\sim) is a precision error that would keep a response from being scored as a three. This response is not a top level response so there is no deduction for a precision error.

In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.



Prove that \overleftrightarrow{XY} and \overleftrightarrow{AC} are parallel.



xy and ac are parallel because of
SAS

► Numbers

► Arithmetic and Units

► Exponents, Roots, Logs

► Relations

► Geometry

► Groups

► Trigonometry

► Statistics

Annotation

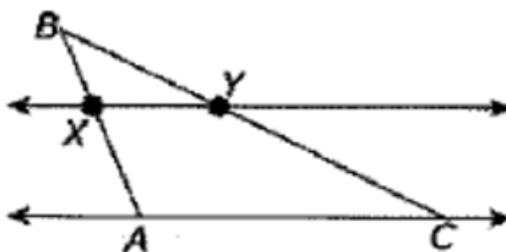
Anchor Paper 7
Score Point 0

The response receives no credit. The student includes none of the required elements.

The response shows an incomplete attempt to construct a proof. The response provides a bald statement with no reasoning for the final statement that is given in the prompt. Therefore, no credit is earned for either element.

In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.



Prove that \overrightarrow{XY} and \overrightarrow{AC} are parallel.



B to X is equal to $\frac{1}{3}$ and B to Y is equal to $\frac{1}{3}$

› Numbers

› Arithmetic and Units

› Exponents, Roots, Logs

› Relations

› Geometry

› Groups

› Trigonometry

› Statistics

Annotation

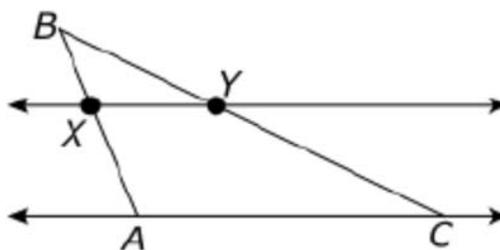
Anchor Paper 8
Score Point 0

The response receives no credit. The student includes none of the required elements.

The response shows no attempt to construct a proof, nor does it provide any justification for its reasoning. Therefore, no credit is earned for either element.

In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.



Prove that \overleftrightarrow{XY} and \overleftrightarrow{AC} are parallel.

Because it is cutting the triangle in half to where there is a smaller triangle on the bottom.

Annotation

Anchor Paper 9
Score Point 0

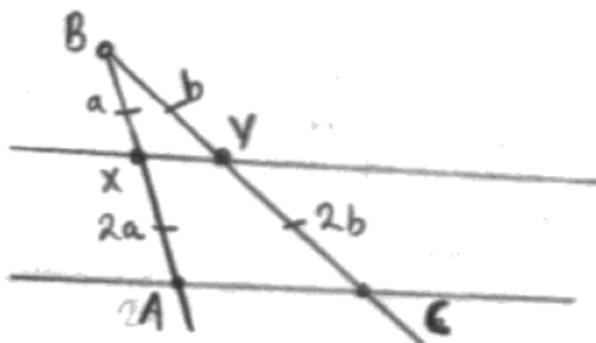
The response receives no credit. The student includes none of the required elements.

The response shows no attempt to construct a proof, nor does it provide any justification for its reasoning. Therefore, no credit is earned for either element.

Practice Set

P1–P10

No Annotations Included

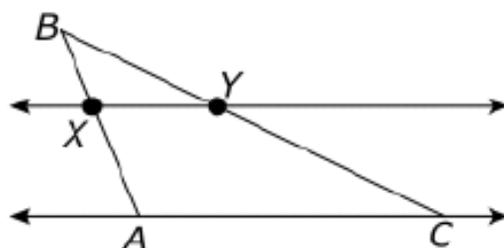


Statement	Reason
① B, X, A and B, Y, C are collinear $BX = \frac{1}{3}BA$ and $BY = \frac{1}{3}BC$	① given
② $\angle ABC = \angle ABC$	② reflexive
③ $\triangle BXY \sim \triangle BAC$	③ SAS \sim
④ $\angle BXY = \angle BAC$	④ definition of similarity (if the Δ s are \sim , their \angle s are =)
⑤ $\overleftrightarrow{XY} \parallel \overleftrightarrow{AC}$	⑤ If $\angle BXY$ and $\angle BAC$ are = corresponding angles, they are \parallel lines

Statements	Reasons
1. $\angle B \cong \angle B$	1. reflexive prop
2. $\frac{1}{3}(BC) = BY$ $\frac{1}{3}(BA) = BX$	2. given
3. $BC \sim BY$ $BA \sim BX$	3. def of \sim
4. $\triangle BXY \sim \triangle BAC$	4. SAS \sim
5. $\angle X \cong \angle A$	5. def of $\sim \triangle s$
6. $\overline{XY} \parallel \overline{AC}$	6. alt corr. $\angle s$ thm.

In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.



Prove that \overleftrightarrow{XY} and \overleftrightarrow{AC} are parallel.

$$\triangle bxy \approx \triangle bac$$

• B, X, A are collinear

B, Y, C are collinear

$$BX = \frac{1}{3}(BA)$$

$$BY = \frac{1}{3}(BC)$$

$$\triangle BXY \sim \triangle BAC$$

$$\angle BXY \cong \angle BAC$$

• \overline{XY} and \overline{AC} are \parallel

• Given

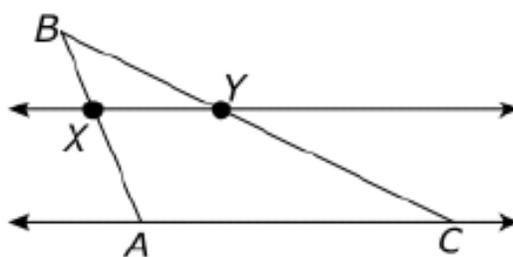
• SS similarity Theorem

• \angle s of similar polygons are \cong

• Congruent corresponding angles \Rightarrow
parallel lines

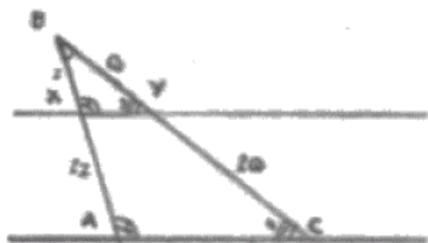
In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.



Prove that \overleftrightarrow{XY} and \overleftrightarrow{AC} are parallel.

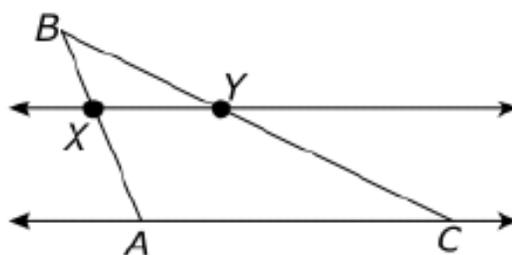
B is collinear with X and A but also Y and C, but the two are not on the same line. Also, since line BX is $\frac{1}{3}$ BA and BY is $\frac{1}{3}$ BC, $\triangle BXY \sim \triangle BAC$



statement	reason
① B, X, and A collinear Y, and C collinear $BX = \frac{1}{3}(BA)$, $BY = \frac{1}{3}(BY)$	① given
② $\angle XBY = \angle XBY$	② reflexive
③ $\triangle XBY \sim \triangle ABC$	③ SAS ~
④ $\angle 1 = \angle 2$, $\angle 3 = \angle 4$	④ cpctc
⑤ \overline{XY} and \overline{AC} are	⑤ when corresponding \angle s of two lines are \cong it implies

In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.

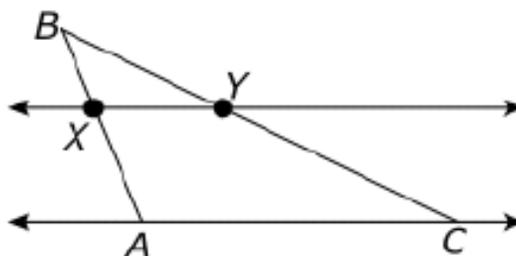


Prove that \overleftrightarrow{XY} and \overleftrightarrow{AC} are parallel.

THEY BOTH ARE THE SAME DISTANCE AND
DONT TOUCH AND GOING THE SAME WAY

In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.



Prove that \overleftrightarrow{XY} and \overleftrightarrow{AC} are parallel.

$$\triangle BXY \sim \triangle BAC$$

$$SAS \sim \square$$

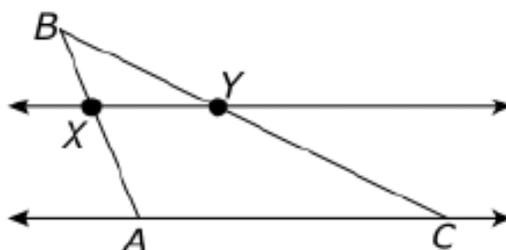
$$\angle BXY \cong \angle BAC$$

CPCTC

If corresponding angles are congruent, then parallel lines.

In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.

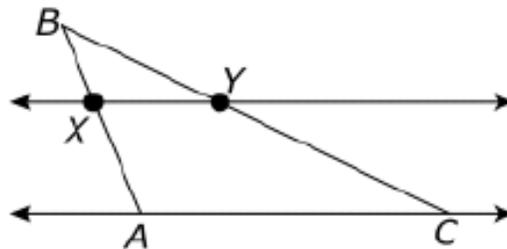


Prove that \overleftrightarrow{XY} and \overleftrightarrow{AC} are parallel.

if you look at the two lines running horizontal it tells you right there that it is

In the given diagram:

- $B, X,$ and A are collinear,
- $B, Y,$ and C are collinear, and
- $BX = \frac{1}{3}(BA)$ and $BY = \frac{1}{3}(BC)$.



Prove that \overleftrightarrow{XY} and \overleftrightarrow{AC} are parallel.

In the diagram there are two triangles given, $\triangle BXY$ and $\triangle BAC$. These triangles are similar because $\angle X$ is congruent to $\angle A$ and $\angle Y$ is congruent to $\angle C$.

Practice Set

Paper	Score
P1	3
P2	2
P3	0
P4	2
P5	1
P6	2
P7	0
P8	2
P9	0
P10	1