Math
Released Item 2019

Geometry
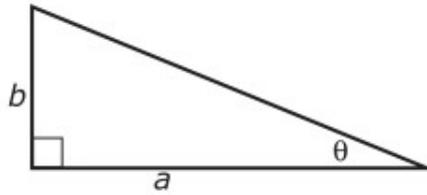
Trish's Right Triangle
M41207

Anchor Set
A1–A8

With Annotations

Prompt

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \leq a \leq 11$ and that $5 \leq b \leq 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

[Math editor toolbar with buttons: refresh, undo, redo, backspace, +, −, ×, ÷, fraction, mixed number, y^x, $\sqrt{}$, =, ≈]

- ▸ Numbers
- ▸ Arithmetic and Units
- ▸ Exponents, Roots, Logs
- ▸ Relations
- ▸ Geometry
- ▸ Groups
- ▸ Trigonometry
- ▸ Statistics
- ▸ Greek

Rubric

	#3 M41207 Rubric
Score	**Description**
3	Student response includes each of the following 3 elements. • Correct value for θ (24.44° to 30.96°). • Shows valid calculations and reasoning to find 24.44° as the smallest value of θ • Shows valid calculations and reasoning to find 30.96° as the largest value of θ Alternately a student response may earn 1 point for each of the following: • Shows valid reasoning to find 24.44° as the smallest value of θ and 30.96° as the largest value of θ. • Shows valid calculations to find 24.44° as the smallest value of θ and 30.96° as the largest value of θ. Valid and complete calculations can also demonstrate reasoning with no further explanation needed. Sample Student Response: To determine θ, use the tangent and the quotient $\frac{b}{a}$. The smallest estimate of θ comes from the smallest value of b and the greatest value of a. $$\theta \approx \tan^{-1}\left(\frac{5}{11}\right)$$ $$\approx 24.44°$$ The greatest estimate of θ comes from the greatest value of b and the smallest value of a. $$\theta \approx \tan^{-1}\left(\frac{6}{10}\right)$$ $$\approx 30.96°$$ Therefore, θ can vary from **24.44°** to **30.96°**. **Note:** It is acceptable to round a degree measure to the nearest ones place, since the side lengths of the triangle are only given to the nearest ones place.
2	Student response includes 2 of the 3 above elements.
1	Student response includes 1 of the 3 above elements.
0	The response is incorrect or irrelevant.

A1

Score Point 3

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \le a \le 11$ and that $5 \le b \le 6$.
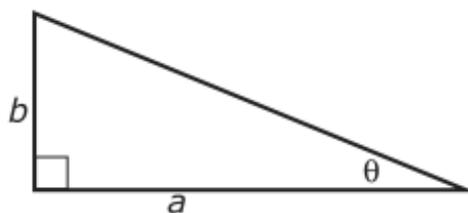
Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

> To get the minimum value of θ, then b must be small and a must be big. So the $Tangent\theta = \frac{b}{a}$ or $\frac{5}{11}$ when you solve for θ you get the minimum is 24.44. To get the maximum θ, b must be big and a must be small. SO $Tangent\theta = \frac{6}{10}$. Smiplified, θ is 30.96 degrees.
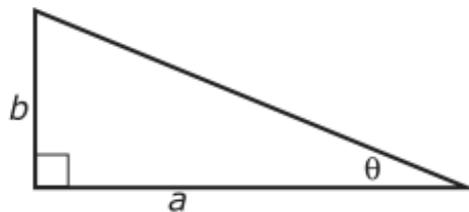
Annotation

This response receives full credit. The student includes all three of the required elements.

- The correct values for the greatest and the least estimate of the degree measure are provided (when you solve for θ you get the minimum is 24.44. To get the maximum θ . . . θ is 30.96 degrees).

- There are valid calculations and reasoning to find the smallest value (To get the minimum value of θ, then b must be small and a must be big. So the Tangentθ = b/a or 5/11 when you solve for θ you get the minimum is 24.44).

- There are valid calculations and reasoning to find the largest value (To get the maximum θ, b must be big and a must be small. So Tangentθ = 6/10. Simplified, θ is 30.96 degrees).

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \leq a \leq 11$ and that $5 \leq b \leq 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

$$\tan^{-1} \frac{b}{a} = \theta$$

Leasts:
$$\tan^{-1} \frac{5}{11} \approx 24$$

Greatests:
$$\tan^{-1} \frac{6}{10} \approx 31$$

Annotation

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \le a \le 11$ and that $5 \le b \le 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

The range is between 24.4 and 31.0. I determined this using inverse tan.

Annotation

Anchor Paper 3
Score Point 2
This response receives partial credit. The response includes two of the three required elements.

- The correct values for the greatest and the least estimate of the degree measure are provided (The range is between 24.4 and 31.0). Note that it is also acceptable to round a degree measure to the nearest tenths place.

- The response shows valid reasoning for finding the lowest and largest degree measure (I determined this by using inverse tan). Note that complete and correct calculations can demonstrate valid reasoning with no further explanation needed.

There are no calculations provided for either value.

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \le a \le 11$ and that $5 \le b \le 6$.
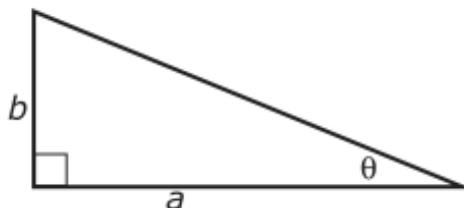
Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

$x \approx 27$ to $29°$

I determined this by taking the least possible number of each letter a being 10 and b being 5, and took the $\tan^{-1}\left(\frac{5}{10}\right)$ and got $27°$ which is the least it could be and took the biggest possible numbers of a $(11) and$ b (6) took $the \tan^{-1}\left(\frac{6}{11}\right)$ which gave me the largest number it could be $29°$.

Annotation

Anchor Paper 4
Score Point 1
This response receives partial credit. The student includes one of the three required elements.

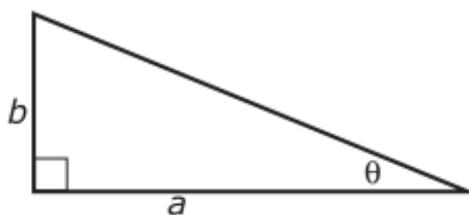- The calculations are correct after an initial error is made in setting up the greatest and least estimates.
  Note: A common mistake is to use the smallest two numbers for the low estimate and the highest two numbers for the high estimate. For this response, the student goes on to correctly calculate the inverse tangent for each value ($x \approx 27$ to $29°$).

The given values are incorrect. The reasoning is incorrect [I determined this by taking the least possible number of each letter *a* being 10 and b being 5. . . and took the biggest numbers of a (11)and b (6)].

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \leq a \leq 11$ and that $5 \leq b \leq 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

$$\tan x = \frac{5}{10}$$
$$x = 26.6$$
$$\tan x = \frac{6}{11}$$
$$x = 28.6$$
$$26.6 \leq \theta \leq 28.6$$

Annotation

Anchor Paper 5
Score Point 1
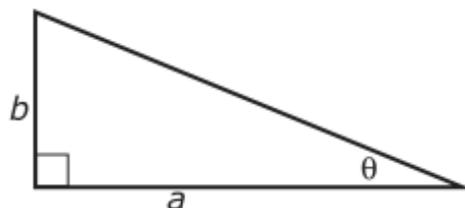This response receives partial credit. The student includes one of the three required elements.

- The calculations are correct after the error in setting up the greatest and least estimates (tan x = 5/10 x = 26.6, tan x = 6/11 x = 28.6). Note: The student incorrectly uses the smallest two numbers for the low estimate and the highest two numbers for the high estimate, but then follows through with correct calculations.

The given values are incorrect due to the incorrect reasoning.

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \le a \le 11$ and that $5 \le b \le 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

$Greatest \approx 31$ degrees
$Least \approx 22$ degrees
I found these estimates by taking the tangent inverses of the highest and lowest possible values for a and b.

Annotation

Anchor Paper 6
Score Point 1
This response receives partial credit. The student includes one of the three required elements.

- The response shows valid reasoning, finding the lowest and largest degree measure (I found these estimates by taking the tangent inverses of the highest and lowest possible values for a and b). This reasoning is further supported since the greatest value is correct (31 degrees),

No credit is awarded for the first element since the least degree measure is incorrect (22 degrees). There are no calculations for finding the degree measures provided.

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \le a \le 11$ and that $5 \le b \le 6$.
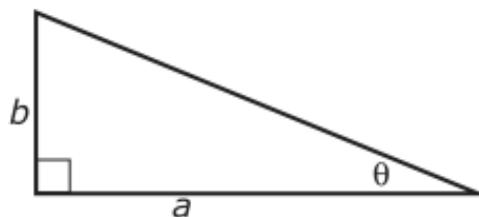
Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

$$\tan^{-1} \theta = \frac{5}{10}$$
$$\tan^{-1} \theta = \frac{6}{11}$$

Annotation
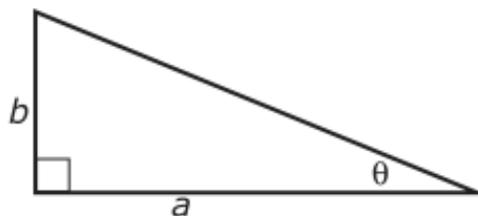
Anchor Paper 7
Score Point 0
This response receives no credit. The student provides none of the required elements.

The correct values are not determined. The set-up is incorrect and there are no calculations made. There is no reasoning provided.

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \leq a \leq 11$ and that $5 \leq b \leq 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

$$\tan^{-1} \frac{b}{a}$$

Annotation

Anchor Paper 8
Score Point 0
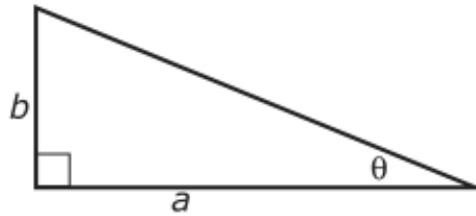This response receives no credit. The student provides none of the required elements.

The correct values are not determined and there are no calculations made. There is no reasoning provided.
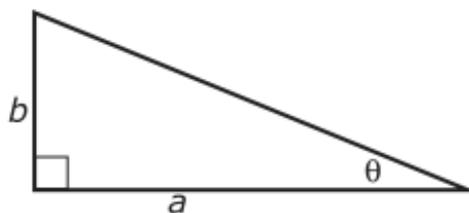
Practice Set
P1–P10
No Annotations Included

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \le a \le 11$ and that $5 \le b \le 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

greatest:30
least:20

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \le a \le 11$ and that $5 \le b \le 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.
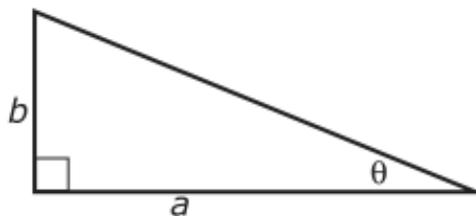
$$\tan^{-1} \frac{6}{11} = .499$$
$$\square \approx 50°$$
$$\tan^{-1} \frac{5}{10} = .463$$
$$\square \approx 46°$$

I determined these estimates because i substituted 10 for a and 5 for b and solved using the tan. I aslo substituted 11 for a and 6 for b, then using the tan to get the degree measures.
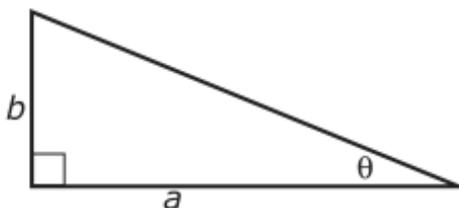
In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \le a \le 11$ and that $5 \le b \le 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

The angle measure of θ has to be between 46 degrees and 50 degrees. I found this out by using $\tan^{-1}$ ☐ on both the lowest numbers and the highest numbers. This gives me the range of the degree measurment.

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \le a \le 11$ and that $5 \le b \le 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.
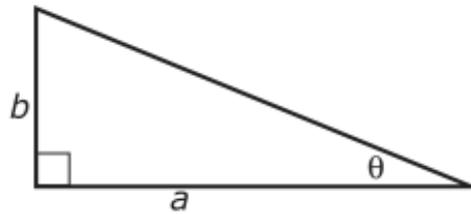
$Least = 26.57°$
$Greatest = 28.61°$

☐

I found this by taking the $\tan^{-1}$ ☐ of θ using the smallest numbers (5) and (10), then using the greatest numbers possible (6) and (11)
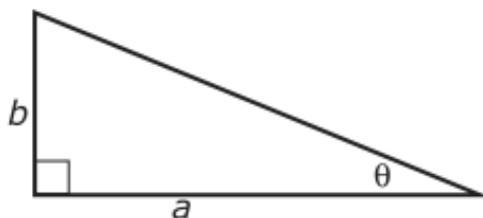
In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \leq a \leq 11$ and that $5 \leq b \leq 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

I estimate that the angle measure of angle 0 is anywhere between 30 and 45 degrees
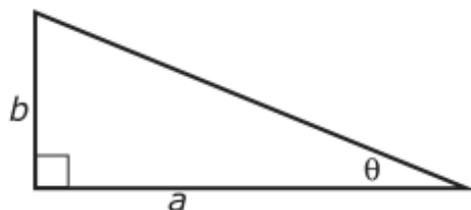
In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \leq a \leq 11$ and that $5 \leq b \leq 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

The degree measure for angle theta can befound by using the inverse tangent of the angle theta. If tangent of theta equals at least $\frac{5}{11}$ and at greatest $\frac{6}{10}$ then the angle thetas greatest and least possible measures can be found by using these fractions by evaluating the functions tangent ot the negative one of each fraction, it was found that the greatest possible angle measure for theta was 3.96 degrees and the least possible measure was 24.44 degrees.

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \leq a \leq 11$ and that $5 \leq b \leq 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

$$\tan^{-1}\left(\frac{10}{5}\right) = \theta$$

$\theta \approx 63.4$ degrees
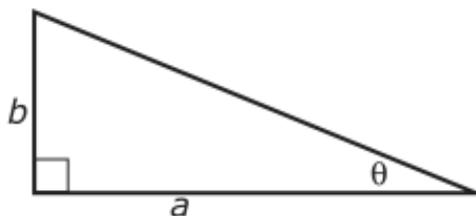
$$\tan^{-1}\left(\frac{11}{6}\right) = \theta$$

$\theta \approx 61.4$ degrees

$61.4 \leq \theta \leq 63.4$

This is because the value of θ is going to be the $\tan^{-1}\ of$ the given $\frac{a}{b}$. This means that $\frac{a}{b}$ must either be $\frac{10}{5}$ or $\frac{11}{6}$, in which these values are the greatest and least values for the degree measure $of\theta$.

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \leq a \leq 11$ and that $5 \leq b \leq 6$.
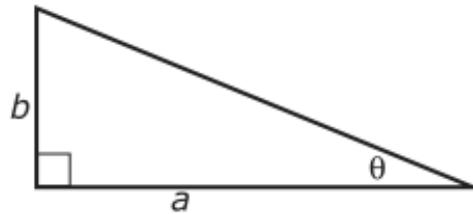

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

$$\tan x = \frac{10}{5} \leq \theta \leq \tan x = \frac{11}{6}$$

so the angle has to be higher than 61.4 degrees but less than 63.4 degrees

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \le a \le 11$ and that $5 \le b \le 6$.

Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

θ is between 24.44 degrees and 30.96 degrees. because the tangent of θ is $\frac{b}{a}$ and to have a max for θ you have to use 6 for b

In the right triangle shown, Trish wants to estimate the degree measure of θ. She knows that $10 \le a \le 11$ and that $5 \le b \le 6$.
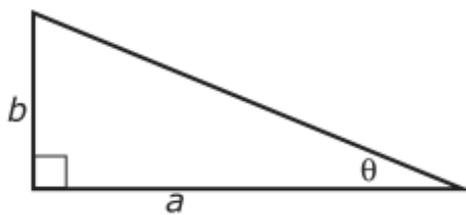
Determine the greatest and least estimates of the degree measure of θ. Explain how you determined each estimate.

$\theta = 30°$

Practice Set

Paper	Score
P1	0
P2	0
P3	0
P4	1
P5	0
P6	2
P7	0
P8	0
P9	2
P10	0